Audio Engineering Society
Convention Paper
Presented at the 111th Convention
2001 September 21–24 New York, NY, USA

Modal Improved Condenser Microphone

Richard Barnert
AKG-Acoustics GmbH
Lemboeckgasse 21-25
A-1230 Vienna, Austria
BarnertR@AKG-Acoustics.com

ABSTRACT

Techniques are shown which make it possible to alter the frequency response and therefore the sound of a condenser microphone by using the nature of diaphragm modal shapes. The modal behaviour can further be influenced by stretching the diaphragm at certain points, which leads to increased sensitivity. By applying these low cost methods, it is possible to modify specific frequency responses and to improve the signal-to-noise ratio in an easy way.

INTRODUCTION

Different recording applications require different types of microphones. Frequency response and sensitivity are two of the most important attributes when choosing the right tool. This paper shows methods to modify and to improve these two attributes of a condenser microphone. The first method is based on the interaction of modal diaphragm shapes and the charge distribution upon the backplate. A mathematical solution is derived from the two-dimensional wave equation. Results of measurements are shown. The second method presented is similar to the first one and uses almost the same equations, but here the goal is a different one. By stretching the diaphragm towards the backplate and fixing it there, increased sensitivtiy can be achieved. This is explained with a mathematical model. Measurements of an existing microphone using this technique are shown as well.

ALTERING THE FREQUENCY RESPONSE

The dynamic properties of a condenser microphone can be described by the modal superposition of its membrane eigenmodes. The shape of the eigenmodes and the corresponding frequencies can be calculated by solving the two-dimensional wave-equation (which is done in the following chapter). They can also be calculated with the help of numerical finite element (FEM) and/or boundary element (BEM) methods [1]. With BEM it is also possible to consider the surrounding medium, while FEM is only valid for vacuum. Unfortunately, both methods are not powerful enough to consider important parameters like acoustic frictions and small voluminas, which determine the real properties of the capsule and influence the actual behaviour of the diaphragm. However, with the help of FEM it is possible to find a practical approximation to reality. A third possibility to discover eigenmodes are optical measurements with a laser vibrometer.

Results of finite element simulation

These calculations are solved numerically. The engineer has to provide the computer with the geometrical and physical properties of his mechanical model. In the case shown here, a one-inch polycarbonate membrane in vacuum is studied (coupled BEM analysis delivers slightly lower frequencies due to air damping and is not discussed in this paper). Figure 1 shows the first and the fourth eigenmode of a diaphragm, which is fixed only at the rim. Figure 2 shows the first and the eigth eigenmode of a diaphragm, which is additionally fixed at the center. The latter construction is also well known as part of the dual-diaphragm “Braunmühl-Weber” design [2]. It must be emphasized, that in the Braunmühl-Weber design the distance between the diaphragm and the backplate is constant, i.e. they are parallel to each other in the unloaded case.

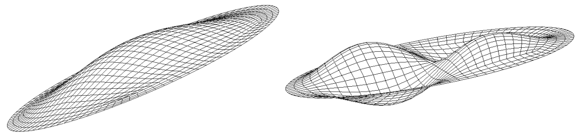

Fig. 1. Simulated Y_{01} and Y_{21} modes of a rim-fixed membrane

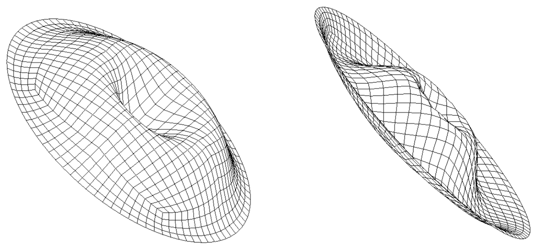

Fig. 2. Simulated Y_{01} and Y_{02} modes of a center-fixed membrane

Figure 3 shows a table with the calculated frequencies for both designs with the corresponding modes Y_{mn} (the integer *m* determines the number of *radial nodal lines* and the integer *n* determines the number of *azimuthal nodal circles*.). Although the fixation in the middle of the membrane forces another azimuthal nodal circle additionally to that at the rim, the index *n* here in this figure was not incremented in order to keep the results comparable.

	Rim-fixed design		Center-fixed design	
Nr.	f [Hz]	m n	f [Hz]	m n
1	731	0 1	1660	0 1
2	1539	1 1	1724	1 1
3	1539	1 1	1724	1 1
4	2536	2 1	2536	2 1
5	2588	2 1	2588	2 1
6	2936	0 2	3808	3 1
7	3806	3 1	3808	3 1
8	3806	3 1	4734	0 2

Fig. 3. Comparison of FEM-calculated eigenmodes

As expected, the center-fixed design delivers a higher fundamental eigenfrequency compared to the other design. For real-world applications this can be adapted by lowering the tension of the diaphragm.

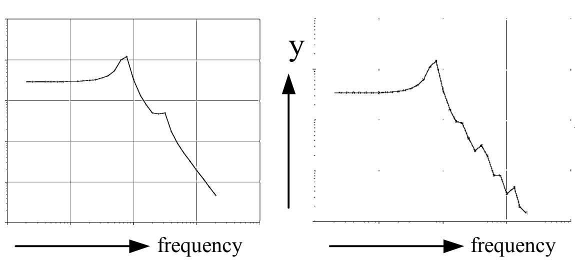

Fig. 4. FEM-calculated membrane displacement vs. frequency

Modal superposition of the calculated eigenmodes leads to figure 4, where the normalized frequency responses of the membrane for both designs are shown. To achieve a diaphragm movement, the membranes were loaded with a uniformly distributed pressure of 3mN/mm². The left figure shows the rim-fixed, the right figure shows the center-fixed design. Here the tension of the center-fixed membrane has been lowered, so that the fundamental frequency of both designs was the same.

Results of laser measurements

For further comparisons, it is possible to perform optical measurements with a laser vibrometer. Due to physical imperfections the mounted diaphragm does not behave exactly as in the mathematical model. The optical measurement is a suitable method to verify the results of the FEM simulation. Figure 5 shows the measured Y_{01} and Y_{11} mode of a rim-fixed membrane in air, figure 6 shows the Y_{02} and Y_{21} mode. While the modes in figure 5 look almost perfect, the other modes in figure 6 reveal the preferential direction of the used polycarbonate film. Imperfections like these should be considered when the mode-dependent output voltage is calculated or the modal modified frequency response is measured (see further below).

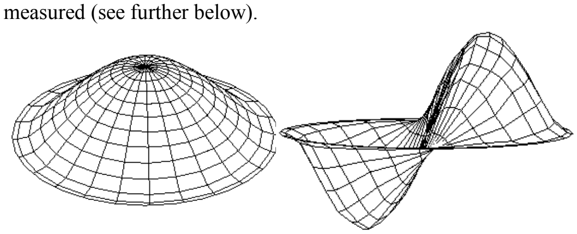

Fig. 5. Measured Y_{01} and Y_{11} mode of a rim-fixed membrane

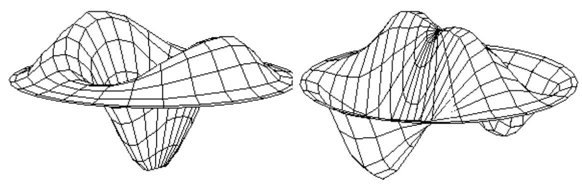

Fig. 6. Measured Y_{02} and Y_{21} mode of a rim-fixed membrane

Output voltage of condenser capsules

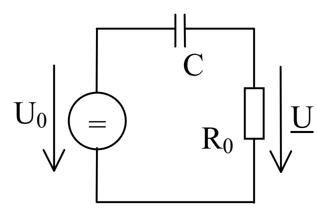

Fig. 6. Low-frequency circuit of a condenser microphone

When the low-frequency circuit like in figure 6 is used, the output voltage $\underline{U} = U\cdot e^{j\omega t}$ is taken from the resistor R_0, which has usually many Mega-Ohms. $\underline{y} = y\cdot e^{j\omega t}$ is the movement of the diaphragm. When *C* is the capacity of the condenser, *d* is the distance of the unloaded condenser plates and $j\omega CR_0 >> 1$, then the output voltage corresponds to [3][4]

$$\underline{U} = U_0 \cdot \frac{\underline{y}}{d} \quad (1)$$

(1) is easy to apply for a piston-like movement of the diaphragm. But our observations from the FEM analysis show that a piston-like movement does not correspond to the reality. So we have to replace $\underline{y}$ with $\underline{y}(r,\varphi)$, with *y(t)* beeing the movement along the y-axis like shown in figure 7. For this reason the two-dimensional wave equation for a circular membrane has to be solved.

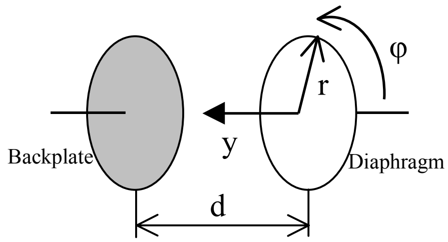

Fig. 7. Condenser diaphragm

General solution of the wave equation

The solution for free vibrations of a circular membrane fixed at the rim can be found in many different books about acoustics and mechanics and is therefore shortened. The two-dimensional wave equation in polar coordinates reads

$$\frac{\partial^2 y}{\partial r^2}+\frac{1}{r}\frac{\partial y}{\partial r}+\frac{1}{r^2}\frac{\partial^2 y}{\partial \varphi^2}=\frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} \quad (2)$$

The harmonic solution may be expressed as

$$y(r,\varphi,t)=R(r)\cdot\Theta(\varphi)\cdot e^{j\omega t} \quad (3)$$

Upon making this substitution, (2) becomes the Helmholtz equation (i.e. the time-independent wave equation) in polar coordinates

$$\Theta\frac{\partial^2 R}{\partial r^2}+\frac{\Theta}{r}\frac{\partial R}{\partial r}+\frac{R}{r^2}\frac{\partial^2 R}{\partial \varphi^2}+k^2 R\Theta=0 \quad (4a)$$

or

$$\frac{r^2}{R}\left(\frac{\partial^2 R}{\partial r^2}+\frac{1}{r}\frac{\partial R}{\partial r}\right)+k^2r^2=-\frac{1}{\Theta}\frac{\partial^2\Theta}{\partial\varphi^2} \quad (4b)$$

where $k=\omega/c$. The left-hand side of this equation, a function of r alone, cannot be equal to the right-hand side, a function of φ alone, unless both functions equal some constant. If we let this constant be m^2, then the right-hand side becomes

$$\frac{\partial^2\Theta}{\partial\varphi^2}=-m^2\Theta$$

which has the harmonic solution

$$\Theta(\varphi)=\cos(m\varphi+\gamma) \quad (5)$$

where γ is the initial phase angle. Because φ is periodic, $y(r,\varphi,t)$ must equal $y(r, \varphi+2\pi, t)$, which restricts the constant m to integral values $m=0,1,2,3...$ Equation (4b) now becomes *Bessel's equation*

$$\frac{\partial^2 R}{\partial r^2}+\frac{1}{r}\frac{\partial R}{\partial r}+\left(k^2-\frac{m^2}{r^2}\right)R=0 \quad (6)$$

Solutions to this equation are the transcendental functions called *Bessel functions* of the *first kind* $J_m(kr)$ and *second kind* $Y_m(kr)$

$$R(r)=A\cdot J_m(kr)+B\cdot Y_m(kr) \quad (7)$$

This is the general solution to (6). We now have to distinguish between our two cases (i.e. fixed in the middle or not). We solve the case of the circular membrane only fixed at the rim first.

Solution for a membrane fixed at the rim

This solution can be found in [5] and [6] among others. The required boundary condition is $y(a, \varphi, t)=0$, if $r=a$ is the radius of the membrane. One has to keep in mind that $Y_m(kr)$ becomes unbounded in the limit $kr\rightarrow 0$. That requires $B=0$ so that

$$R(r)=A\cdot J_m(kr) \quad (8)$$

Application of the boundary condition $R(a)=0$ requires $J_m(ka)=0$. If the values of the argument of the mth order Bessel function J_m which cause that function to equal zero are designated by j_{mn}, then we have $J_m(j_{mn})=0$, so that the allowed values of k assume discrete values given by $k_{mn} = j_{mn}/a$. Values of j_{mn} for some zeros of some of the Bessel functions are given in tables or can be found numerically. The normal modes of vibrations are therefore

$$y_{mn}(r,\varphi,t)=A_{mn}J_m(k_{mn}r)\cdot\cos(m\varphi+\gamma_{mn})\cdot e^{j\omega_{mn}t} \quad (9)$$

where $k_{mn}a=j_{mn}$ and the natural frequencies are $\omega_{mn} = c\cdot j_{mn}/a$.

The physical motion of the eigenmode designated by the integers *(m,n)* is the real part of (9). Like before in figures 1 and 2, the integer m determines the number of *radial nodal lines* and the integer n determines the number of *azimuthal nodal circles*. Corresponding to the results of the FEM analysis and various laser measurements, experience has shown that not all of the calculated eigenmodes do really exist.

Solution for a membrane additionally fixed at the center

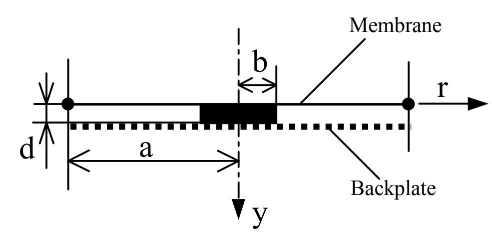

Fig. 8. Fixed membrane

There must be a certain region $0 \leq r \leq b$, where the membrane is fixed, because $Y_m(kr)$ becomes unbounded in the limit $kr\rightarrow 0$. The solution (7) is still valid, but the required boundary conditions are now $y(a, \varphi, t)=0$ and $y(b, \varphi, t)=0$, which leads to

$$J_m(k_{mn}a)\cdot Y_m(k_{mn}b)=J_m(k_{mn}b)\cdot Y_m(k_{mn}a) \quad (10)$$

The allowed values of k again assume discrete values and can be found numerically. Corresponding to (9) this yields

$$y_{mn}(r,\varphi,t)=\left[A_{mn}J_m(k_{mn}r)+B_{mn}Y_m(k_{mn}r)\right]\cdot \cdot\cos(m\varphi+\gamma_{mn})\cdot e^{j\omega_{mn}t} \quad (11)$$

Like before, the physical motion of the eigenmode designated by the integers *(m,n)* is the real part of (11).

Mode-dependent output voltage

In order to get the right output voltage corresponding to the real shape of the diaphragm, one has to replace (1) with

$$U_{mn} = \frac{1}{r^2 \pi} \int_{r=b}^{a} \int_{\varphi=0}^{2\pi} U_0 \frac{\mathrm{Re}\{y_{mn}\}}{d} r\, dr\, d\varphi \qquad (12)$$

with $b=0$ for the rim-fixed membrane and $0 \leq b \leq a$ for a membrane additionally fixed at the center. (12) is valid only for the corresponding frequency ω_{mn} . For the first mode with amplitude y this yields

$$U_{01}(t) = 0.73 \frac{U_0}{d} y \cos(\omega_{01} t) \qquad (13)$$

which offers a remarkable difference from the piston-like diaphragm movement considered in (1).

Practical considerations

It is also possible to go further by making the polarization voltage U_0 dependent on the radius r. Now it is possible to influence the amplitude of certain modes. This can easily be done with electret condenser capsules by modifying the charge distribution upon the backplate. A tool for measuring these distributions was presented at the German VDT International Audio Convention, 2000 November in Hannover [7]. Figure 9 shows a non-uniform charge distribution upon an electret backplate. Figure 10.a shows the frequency response of the capsule for a uniform distribution, figure 10.b shows the response corresponding to the non-uniform distribution of figure 9.

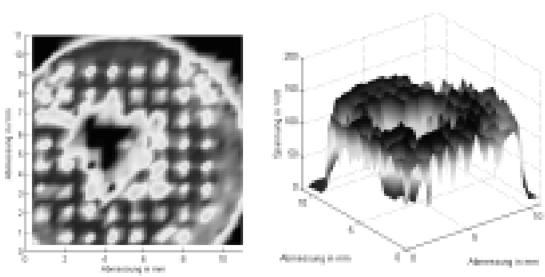

Fig. 9. Non-uniform charge distribution upon a backplate

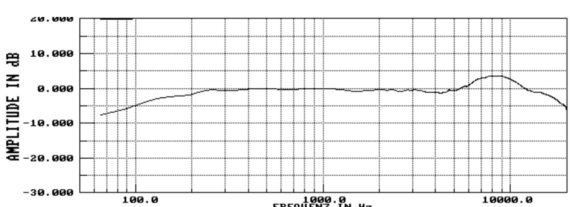

Fig. 10.a. Frequency response corresponding to uniform charge distribution

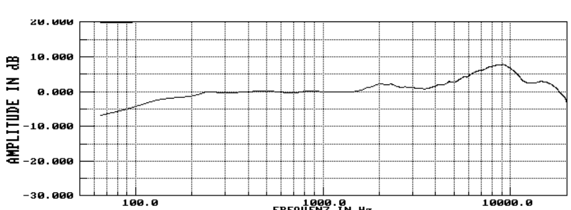

Fig. 10.b. Frequency response corresponding to non-uniform charge distribution of fig.9.

Although this method offers another degree of freedom to the designer, it has an obvious disadvantage. Because condenser diaphragms tend to stick to the backplate when the charge is too high, the backplate must not be "overloaded". This is a limit for the maximum sensitivity which can be achieved. By changing the charge distribution from uniform to non-uniform the maximum sensitivity is likely to be decreased. In order to compensate this expected loss of sensitivity, a method is presented in the next chapter, which helps to increase the sensitivity again.

INCREASING THE SENSITIVITY

The following figure shows an EP patent bending capsule design (EP 1 120 996.A). The diaphragm is stretched and fixed to the center of the backplate. This is a remarkable difference to the well known Braunmühl-Weber design, where the unloaded membrane is supposed to be parallel to the backplate.

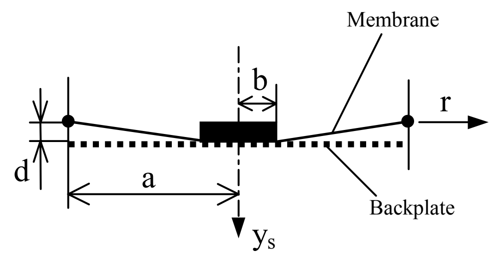

Fig. 11. Stretched membrane

Applying a FEM analysis to that design shows no considerable difference compared to the modes and eigenfrequencies of the Braunmühl-Weber design as long as $d << a$, which is valid for almost all condenser capsules.

Mode-dependent output voltage

In order to get the right output voltage corresponding to the modified shape of the diaphragm, one has to superpose y_{mn} in (12) with the shape of the stretched membrane as shown in fig. 11, which is

$$y_s(r) = \left(1 - \frac{r-b}{a-b}\right) \cdot d \qquad \text{with } r \geq b \qquad (14)$$

This yields

$$U_{mn} = \frac{1}{r^2 \pi} \int_{r=b}^{a} \int_{\varphi=0}^{2\pi} U_0 \frac{\mathrm{Re}\{y_{mn}\} + y_s(r)}{d} r\, dr\, d\varphi \qquad (15)$$

Due to $d << a$, it is not useful to solve the wave equation once again. (15) is close enough for practical purposes. Because the center regions of the diaphragm are closer to the backplate than the outer regions, and the fundamental mode has its maximum elongation near the center (see figure 2), the low frequencies are expected to be boosted.

Practical results

A ½-inch condenser capsule using this patented design is beeing produced since spring 2000. In order to find the maximum sensitivity during the development process, the charge upon the backplate has been constantly increased, until the diaphragm has started to stick to the backplate during temperature tests. In this manner, the maximum charge has been found experimentally. Figure 12 shows the frequency response of one and the same capsule with two different membranes. The upper curve represents the capsule with the stretched membrane, the other curve shows the same capsule – only a different diaphragm was used, which was *neither* stretched *nor* fixed. According to (15) the low frequencies are boosted for the stretched case. However, the maximum sensitivity was increased by 3dB when using the fixed diaphragm design.

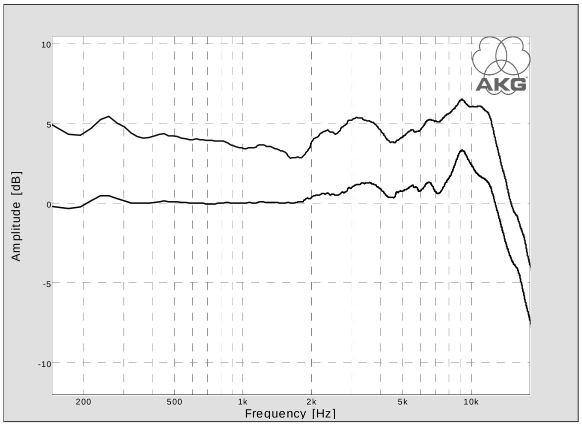

Fig. 12. Comparison of measured frequency responses

SUMMARY

Two methods were presented that help to improve the features of condenser microphones. One method modifies the frequency response, the other method increases the sensitivity. The theoretical background was explained by solving the two-dimensional wave equation, results of simulations were compared to measurements in order to verify the theoretical conclusions. The presented techniques can be applied together or independent from each other and add another degree of freedom to the acoustic design of condenser capsules. Changing the charge distribution upon backplates is a powerful method to create specific frequency responses, while increasing the sensitivity can be very useful, especially for small diaphragm designs in order to improve the signal-to-noise ratio.

REFERENCES

[1] NADwork Theoriehandbuch, Numerical Analysis and Design GmbH & Co KG, 1992-1999

[2] Braunmühl u. Weber, Hochfrequenztechnik und Elektroakustik, 1935

[3] Cremer u. Heckl, Körperschall, Springer-Verlag 1982

[4] Zwicker u. Zollner, Elektroakustik, Springer-Verlag 1993

[5] Kinsler et. al., Fundamentals of Acoustics, John Wiley & Sons, 1982

[6] Morse, Vibration and Sound, Acoustical Society of America, 1995

[7] Barnert et. al., Ladungsverteilung in Elektret-Mikrofonkapseln, VDT International Audio Convention 2001